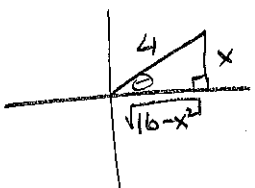
Simplify the following expressions completely. Show proper reasoning to justify your answer. **SCORE: ____ / 12 PTS**

[a] $\cot(\sin^{-1}\frac{x}{4})$, where $x > 0$ $= \frac{\sqrt{16-x^2}}{x}$

$\theta = \sin^{-1}\frac{x}{4}$

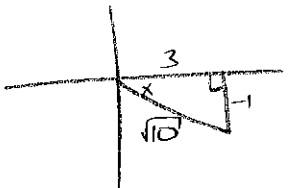
$\sin\theta = \frac{x}{4}$ AND $x \in Q_1$



[b] $\csc(\arctan(-\frac{1}{3})) = -\sqrt{10}$

$x = \arctan(-\frac{1}{3})$

$\tan x = -\frac{1}{3}$ AND $x \in Q_4$



Graph 2 periods of the function $y = -5\cos(\frac{2}{3}x + \frac{5\pi}{3}) - 4$.

SCORE: _____ / 16 PTS

Find the coordinates of the 9 points discussed in lecture, corresponding to 2 complete periods, starting at the phase shift.
Label all $x-$ and $y-$ values for the 9 points on the appropriate axes, using a consistent scale for each axis.

MIDDLE $y = -4$ TOP $= -4+5 = 1$ PERIOD $= \frac{2\pi}{\frac{2}{3}} = 2\pi \cdot \frac{3}{2} = 3\pi$

AMPLITUDE $|-5| = 5$ BOTTOM $= -4-5 = -9$ START $\frac{2}{3}x + \frac{5\pi}{3} = 0$

$\frac{2}{3}x = -\frac{5\pi}{3}$

$x = -\frac{5\pi}{3} \cdot \frac{3}{2} = -\frac{5\pi}{2}$

$\frac{1}{4}$ PERIOD $= \frac{3\pi}{4}$

POINTS:

($\frac{-10\pi}{4}$, -9)

($\frac{-7\pi}{4}$, -4)

($\frac{-4\pi}{4}$, 1)

($\frac{-\pi}{4}$, -4)

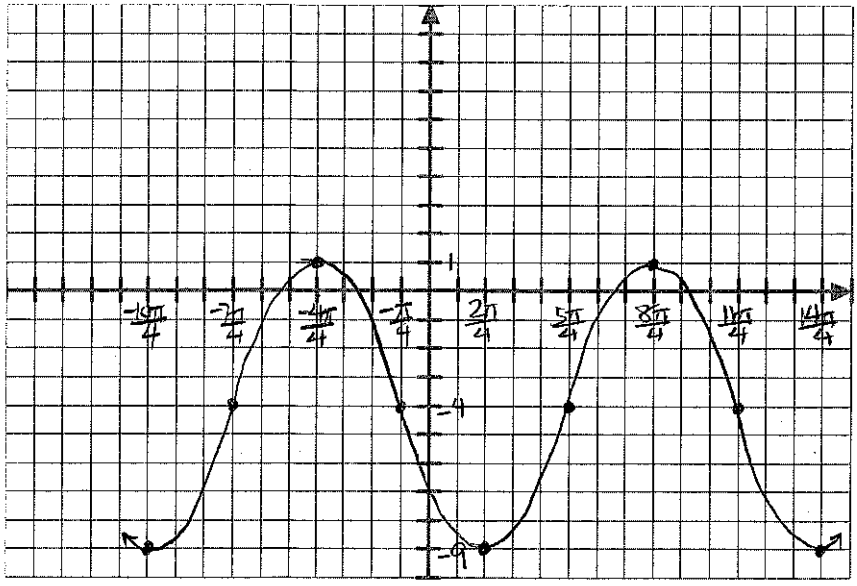
($\frac{2\pi}{4}$, -9)

($\frac{5\pi}{4}$, -4)

($\frac{8\pi}{4}$, 1)

($\frac{11\pi}{4}$, -4)

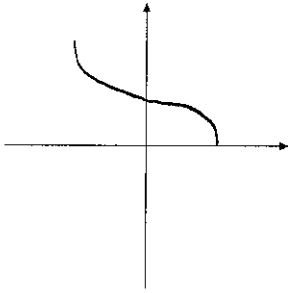
($\frac{14\pi}{4}$, -9)



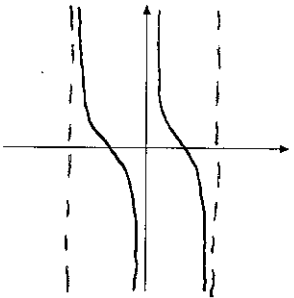
Sketch the graphs. **For periodic functions, sketch at least 2 periods.**
You only need to get the general position and shape correct. Do NOT plot points.

SCORE: ____ / 12 PTS

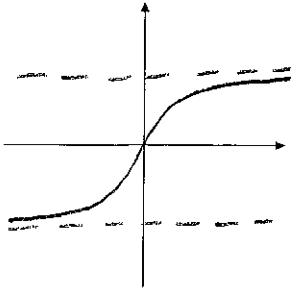
[a] $y = \cos^{-1} x$



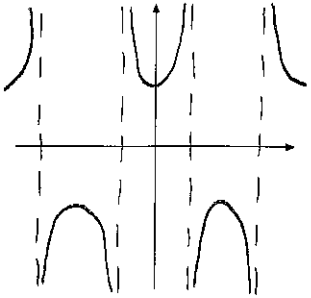
[b] $y = \cot x$



[c] $y = \arctan x$



[d] $y = \sec x$



Fill in the blanks. Write "DNE" if the question has no answer.

SCORE: ____ / 32 PTS

[a] $\arccos(\cos\frac{5\pi}{4}) =$ $\frac{3\pi}{4}$.

[b] $\tan(\arctan 3) =$ 3.

[c] $\sin(\sin^{-1} 3) =$ DNE.

[d] $\cos^{-1}(\cos\frac{5\pi}{6}) =$ $\frac{5\pi}{6}$.

[e] The range of $f(x) = \arccos x$ is $[0, \pi]$.

[f] The domain of $f(x) = \tan x$ is $x \neq \frac{\pi}{2} + n\pi$, $n \in \mathbb{Z}$

[g] As $x \to \frac{\pi}{2}^+$, $\sec x \to$ $-\infty$.

[h] As $x \to 0^-$, $\cot x \to$ $-\infty$.

[i] The domain of $f(x) = \sin^{-1} x$ is $[-1, 1]$.

[j] The range of $f(x) = \sec x$ is $(-\infty, -1] \cup [1, \infty)$

[k] The equations of the asymptotes of $f(x) = \arctan x$ are

$y = \pm\frac{\pi}{2}$.

[l] The equations of the asymptotes of $f(x) = \csc x$ are

$x = n\pi$, $n \in \mathbb{Z}$

[m] $\arctan(-1) =$ $-\frac{\pi}{4}$.

[n] $\cos^{-1}(-\frac{\sqrt{2}}{2}) =$ $\frac{3\pi}{4}$.

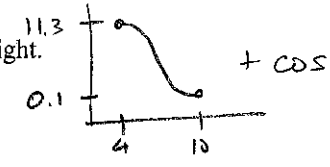
[o] $\arcsin\frac{1}{2} =$ $\frac{\pi}{6}$.

[p] $\tan^{-1}\sqrt{3} =$ $\frac{\pi}{3}$.

The depth of the water at the end of a dock is a sinusoidal function. **SCORE: ____ / 14 PTS**

At 4am, the water has its maximum depth of 11.3 feet, and the depth decreases until, and at 10am, the water has its minimum depth of 0.1 feet.

[a] Find an equation for the depth of the water at t hours after midnight.



MIDDLE $= \frac{11.3+0.1}{2} = 5.7 = D$

AMPLITUDE $= \frac{11.3-0.1}{2} = 5.6 = |A|$

PERIOD $= 2(10-4) = 12 = \frac{2\pi}{B} \rightarrow B = \frac{2\pi}{12} = \frac{\pi}{6}$

START $= 4$

$5.6 \cos \frac{\pi}{6}(t-4) + 5.7$

[b] Find the depth of the water at 2pm. (Round your answer to 1 decimal point.)

$t = 12+2 = 14$

$5.6 \cos \frac{\pi}{6}(14-4) + 5.7$

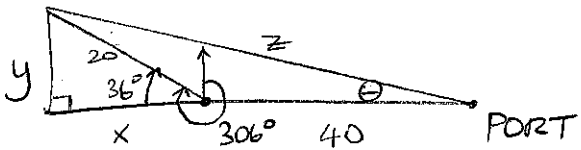
$= 5.6 \cos \frac{5\pi}{3} + 5.7$

$= 5.6\left(\frac{1}{2}\right) + 5.7 = 8.5$ FT

A ship leaves port at noon and heads due west at 20 nautical miles per hour. **SCORE: ____ / 14 PTS**

At 2pm, the ship changes course to a bearing of 306°.

[a] How far is the ship from port at 3pm ? (Round your answer to the nearest integer.)



$\cos 36^\circ = \frac{x}{20} \rightarrow x = 20\cos 36^\circ \approx 16.18$

$\sin 36^\circ = \frac{y}{20} \rightarrow y = 20 \sin 36^\circ \approx 11.76$

$z = \sqrt{(40+16.18)^2 + 11.76^2} \approx 57.4$

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[b] What is the bearing of the ship from port at 3pm ? Your final answer should be a single number (ie. "bearing of _____ °"). (Round your answer to the nearest degree.)

$\tan\theta = \frac{11.76}{40+16.18} \approx 0.2093$

$\theta \approx \tan^{-1}(0.2093) \approx 12^\circ$ BEARING $\approx 270^\circ + 12^\circ = 282^\circ$